

FELO: A ONE-DIMENSIONAL TIME-DEPENDENT FEL OSCILLATOR CODE

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Abstract

A one-dimensional, SDDS compliant time-dependent FEL oscillator code has been developed in Fortran 90. The code, FELO, solves universally-scaled FEL equations to simulate oscillator FELs operating from the low to high gain regime. The code can simulate start-up from shot noise, different electron pulse current distributions, the effects of cavity length detuning and temporal jitter between electron bunches. Cavity detuning curves for both the low-gain IR-FEL and the regenerative amplifier VUV-FEL of the 4th Generation Light Source (4GLS) proposal at Daresbury Laboratory are modelled. The code predictions for the VUV-FEL output are compared with simulations performed with the parallel implementation of Genesis 1.3 and are found to be in good agreement.

INTRODUCTION

Several computational codes are available to the FEL community to aid with the simulation and design of FEL amplifiers e.g. Genesis 1.3 [1]. The authors have developed a simulation package containing FEL simulation code and pre and post-processors that models a FEL with cavity feedback. The code, called FELO, solves a one-dimensional spatio-temporal set of equations which are able to model much of the physics relevant to such FELs. The design of two of the UK 4GLS project FELs were aided by FELO and examples from the 4GLS Conceptual Design Report [2] are presented.

THEORETICAL MODEL

The 1D equations governing the spatio-temporal evolution of the pulsed FEL interaction are well known and in the the universal scaling of [3, 4] may be written:

$$\frac{d\theta_j}{d\bar{z}} = p_j \quad (1)$$

$$\frac{dp_j}{d\bar{z}} = -(A(\bar{z}, \bar{z}_1) \exp(i\theta_j) + c.c.) \quad (2)$$

$$\left(\frac{\partial}{\partial \bar{z}} + \frac{\partial}{\partial \bar{z}_1} \right) A(\bar{z}, \bar{z}_1) = \chi(\bar{z}_1) b(\bar{z}, \bar{z}_1) \quad (3)$$

where $\theta_j = (k + k_u)z - \omega t_j$ is the phase of the j th electron with respect to the particular ponderomotive potential well in which it evolves and $p_j = (\gamma_j - \gamma_r) / \rho \gamma_r$ is its scaled energy, $b(\bar{z}, \bar{z}_1) \equiv \langle e^{-i\theta(\bar{z})} \rangle_{\bar{z}_1}$ is the bunching factor and is the average over the electrons contained within the ponderomotive well centred at \bar{z}_1 at distance through

the interaction region \bar{z} . The weight factor $\chi(\bar{z}_1) = I(\bar{z} = 0, \bar{z}_1) / I_{pk}$ where $I(\bar{z} = 0, \bar{z}_1)$ describes the electron pulse current distribution of peak value I_{pk} at the entrance to the FEL interaction region and $A(\bar{z}, \bar{z}_1)$ is the scaled complex radiation field envelope with magnitude defined by the radiation and electron beam powers as $|A|^2 = P_{rad} / \rho P_{beam}$.

The scaled independent variables may be written [4]:

$$\bar{z} = \frac{z}{l_g} = 2\rho k_u z \quad (4)$$

$$\bar{z}_1 = \frac{z - c\bar{\beta}_z t}{l_g (1 - \bar{\beta}_z)} \quad (5)$$

where $l_g = \lambda_u / 4\pi\rho$ is the nominal gain length of the FEL interaction, the initial z-component of the mean electron velocity within the interaction region is $c\bar{\beta}_z = \langle v_{z0} \rangle$ and ρ is the fundamental FEL, or Pierce, parameter [3]. The latter may be written in practical units for a planar undulator as $\rho \approx 5.7 \times 10^{-3} \gamma_r^{-1} (I_a^2 \lambda_u^2 f_B^2 / \sigma_b^2)^{1/3}$ where f_B is the usual difference of Bessel function factor associated with planar undulators and σ_b is the RMS electron beam radius which for an electron beam of normalised emittance ϵ_n in a focussing channel of beta-function β is given by $\sigma_b = \sqrt{\epsilon_n \beta / \gamma_r}$.

For the case of small gain FELs typical to those encountered in high-Q cavity FELs, previous studies [5] have shown that the effects of emittance and energy spread, electron/radiation pulse slippage effects and the relative transverse overlap between electron beam and cavity modes may be accounted for in correction factors to the expressions for the FEL gain. A small signal gain coefficient was defined which in the universal scaling of the above equations may be written:

$$g_0 = \frac{\bar{z}^3}{\pi} \quad (6)$$

The maximum single pass gain is then approximated by the fitting formula:

$$G_{max} = g_0 F (0.85 + g_0 F (0.19 + 4.12 \times 10^{-3} g_0 F))$$

where the factor $F = F_{inh} F_f F_c$ is a product of correction factors that that respectively account for the effects of inhomogeneous broadening due to emittance and energy spread; a filling factor for the transverse overlap; and the effects of relative slippage between the radiation and electron pulses. The inhomogeneous broadening factor is given by:

$$F_{inh} = \frac{1}{(1 + 1.7\mu_\gamma^2)(1 + \mu_\epsilon^2)} \quad (7)$$

where $\mu_\gamma = 4N_u\sigma_\gamma/\gamma$ and $\mu_\epsilon = 2N_u\epsilon_n\bar{a}_u/\lambda_u(1 + \bar{a}_u^2)$. The filling factor is given by:

$$F_f = \frac{1}{1 + \bar{w}^2/4\sigma_b^2} \quad (8)$$

where \bar{w} is the mean optical mode size, defined by the radius at which the intensity drops to $1/e^2$ of its on-axis value, averaged over the length of the undulator.

Noting from (4 & 6) that $g_0 \propto \rho^3$ and that the expression for G_{max} contains only factors $g_0 F$ allows the inhomogeneous fitting factor F_{inh} and that for the filling factor F_f to be incorporated into the definition of an effective FEL parameter

$$\rho_{eff} \equiv (F_{inh}F_f)^{1/3} \rho \quad (9)$$

to replace that of ρ in the definitions and working equations of (2.5). Note that the factor that accounts for slippage effects, F_c , is not used in the definition of ρ_{eff} as slippage is directly modelled by the partial derivatives of the wave equation (3). It is interesting to note that from the definitions of g_0 and the filling factor F_f that the product $g_0 F_f$, can be expressed as

$$g_0 F_f \propto \frac{1}{\sigma_b^2 + \bar{w}^2/4}.$$

Hence, if the electron beam radius $\sigma_b \ll \bar{w}$, as for example in a long wavelength FEL, the gain is independent of the electron beam radius.

When used with the ρ_{eff} scaling the above equations (2.3) will then estimate the effects of inhomogeneous broadening due to electron energy spread and emittance, and also the effects of transverse electron-radiation coupling, without the need for increased number of computational particles required in 2-D and 3-D models. The FELO code which uses these equations is therefore significantly faster to run than these codes and yet, as will be shown in the following work, gives results which are in good agreement with 3-D simulations.

COMPUTATIONAL MODEL

The working equations (2.3) are solved using the method of characteristics as described in [6] using a code written in Fortran-90. The method of characteristics allows the resultant equations to be integrated via a 4th order Runge-Kutta method with slippage effects between electrons and radiation modelled using simple array shifts. Because the effects of electron energy spread, emittance and transverse coupling are simply accounted for in the modified FEL parameter ρ_{eff} , no electron macroparticle distribution in the scaled energy parameter p is required and $p_j = 0 \forall j$ to describe a resonant FEL interaction. Thus, only a relatively small number of macroparticles are required per ponderomotive well (typically ~ 100) distributed uniformly in θ . The option to include shot-noise effects is included using the method of [7]. Although other shot-noise models more correctly describe the physics [8],

the method of [7] is sufficient here. The FEL cavity length can be varied to allow investigation of cavity detuning effects and the effects of a random temporal jitter in the electron arrival into the cavity can be simulated by introducing a small random variation about the mean cavity length [9].

The FELO source code is written in Fortran-90 and is freely available [10]. The code has been tested using the open-source g95 compiler [11]. A parameter spreadsheet is available to assist the user in preparing the relatively simple input file for the code. Output from the code is written into SDDS formatted files [12]. Post-processing and plotting routines are provided using both SDDS toolkit functions in Tcl script files and MATLAB and are described in more detail in the user-manual provided for the code.

EXAMPLE SIMULATIONS

The FELO code has been used in the design of two of the three FELs of the UK 4GLS project [2]. Both the IR-FEL and the VUV-FEL are cavity FELs. The IR-FEL is designed to operate in the wavelength range 2.5-200 μm . It has a relatively low gain and needs a relatively high Q cavity to lase. The VUV-FEL is designed to generate photon energies of 3-10 eV and will operate in the intermediate gain regime ($\bar{z} \sim 4$) where the FEL interaction evolves exponentially but cannot achieve saturation without seeding in a single pass. Saturation is made possible by introducing a small amount of feedback via a low Q cavity [13]. This also has the advantage of cleaning up the spectral quality of the output over that of SASE [14]. FELO simulations for both of these designs are presented and, in the case of the VUV-FEL, comparison is made with the results of the 3-D code Genesis 1.3 [1]. No such comparison is available for the IR-FEL due to the large number of cavity round-trips required to achieve saturation and the prohibitively long simulation time this would require.

4GLS IR-FEL

The IR-FEL simulations presented are for 2.5 μm operation of the IR-FEL. From the detailed specifications of [2], with an electron energy of 60 MeV, charge 200 pC and RMS duration of 2 ps the IR-FEL operates at 2.5 μm . The undulator has 50 periods, so that the approximate efficiency $\eta \approx 1/4N_u$ gives a peak power output at saturation of $P_{pk} \approx 12$ MW. A more detailed estimate of the efficiency that accounts for passive cavity losses due to mirror reflectivity and all diffraction losses has been derived and reduces the efficiency from that above [2]. Including these effects gives the optimised power estimate for operation of the IR-FEL from 2.5-25 μm as shown in Fig. 1. A FELO simulation at 2.5 μm operation with cavity length detuning of $\delta_c = 9 \mu\text{m}$, so that the output pulse width is close to its maximum, gives good agreement with these estimates as shown in Fig. 2. (Note that positive values of δ_c correspond to a shortened cavity length.) Fig. 3 plots the results of FELO simulations for both the peak power and FWHM

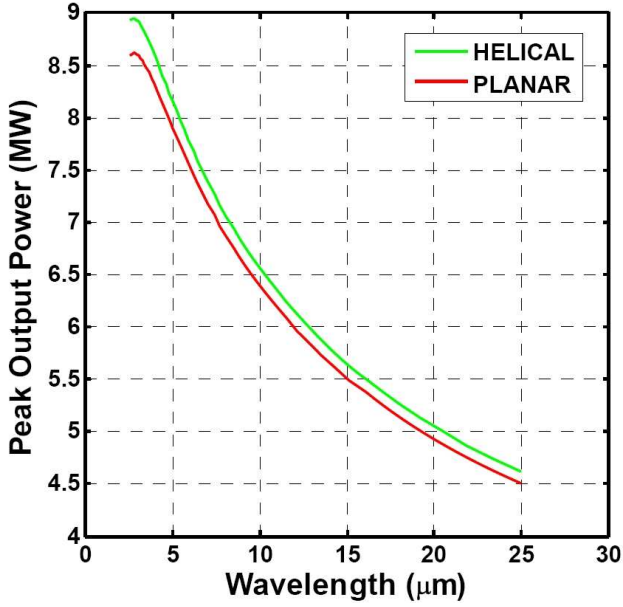


Figure 1: The peak output power for the 4GLS IR-FEL, assuming 2 ps RMS electron pulses and an outcoupling fraction optimised for every wavelength to give the maximum output power. Calculations for helical and planar mode are shown in green and red respectively.

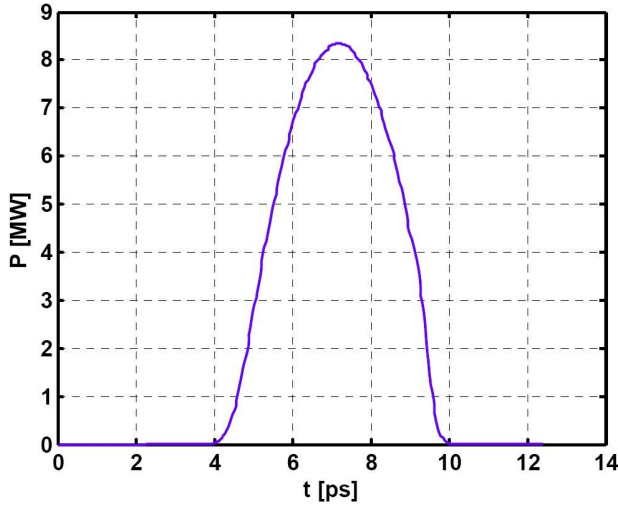


Figure 2: 4GLS IR-FEL pulse power as a function of time for 2.5 μm operation with a cavity length detuning of 9 μm .

pulse width for 2.5 μm operation as a function of cavity detuning δ_c . As cavity resonance is approached at $\delta_c = 0$ it is seen that the peak output power increases and pulse width decreases, typical of superradiant behaviour in an FEL cavity [15].

4GLS VUV-FEL

FELO simulations have also been used in the conceptual design of the 4GLS VUV-FEL. This FEL operates with an intermediate gain of $\bar{z} \sim 4$ and uses a low Q cavity.

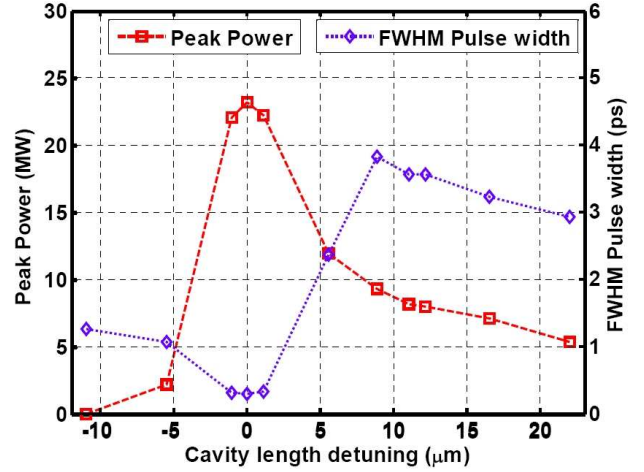


Figure 3: Peak power and FWHM pulse width as a function of cavity length detuning, for the 4GLS IR-FEL at 2.5 μm with 2 ps electron bunches.

Saturation then occurs after only a few cavity passes, typically ~ 10 , which allows multi-pass simulation to saturation using the 3-D code Genesis. For 10 eV operation in a planar undulator, the VUV-FEL conceptual design uses a 600 MeV, 300A peak current electron beam and gives an FEL parameter of $\rho \approx 1.5 \times 10^{-3}$. Although utilising cavity feedback, the VUV-FEL still saturates as a high-gain FEL [13] so that the efficiency is $\eta \approx \rho$. Hence, the saturated power can be approximated as $P_{\text{sat}} \approx \rho P_{\text{beam}} \approx 270$ MW. Fig. 4 plots the results of FELO simulations for both the peak power and FWHM pulse width for 10 eV photon energy operation as a function of cavity detuning δ_c . There are clear similarities with the IR-FEL cavity detuning curve of Fig. 3. As with the IR-FEL, assuming the greatest pulse width approximates most closely to the steady-state, the saturated power for cavity length detuning of $\delta_c \approx 18 \mu\text{m}$ agrees well with the saturated power estimate of ≈ 270 MW of above. The plot of Fig. 5 shows a comparison of the pulse power as the VUV-FEL pulse energy saturates for the case of a FELO (blue) and Genesis (red) simulation for a cavity detuning of 12 μm . The number of cavity passes for the Genesis simulation was 8 while that for the FELO was 30. Although the FELO simulation approached pulse energy saturation after 8 cavity passes, the shape of the pulse power underwent a transition from a narrower, higher peak power pulse to that shown. This accounts for the higher peak power and narrower pulse width of Fig. 4 where $\tau_{\text{FWHM}} \approx 65$ fs $\Rightarrow s \approx 20 \mu\text{m}$ in the units of Fig. 5. This effect of higher peak power and narrower pulse width of the FELO simulation appears to be a transitory superradiant phase. It is not thought that the Genesis simulations show this behaviour, although this will need to be investigated further. As cavity resonance is approached, the radiation pulse width shortens to approximately an order of magnitude less than the electron pulse width of 250 fs FWHM (75 μm). Also, pulse

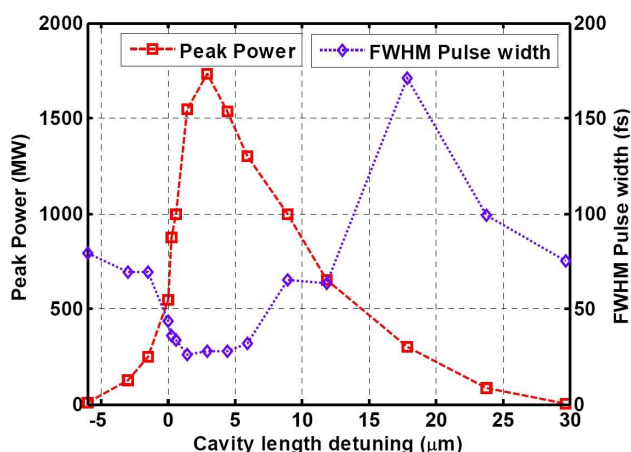


Figure 4: Peak power (red) and FWHM pulse width (blue) as a function of VUV-FEL cavity detuning. The parameters are for 10 eV photon output.

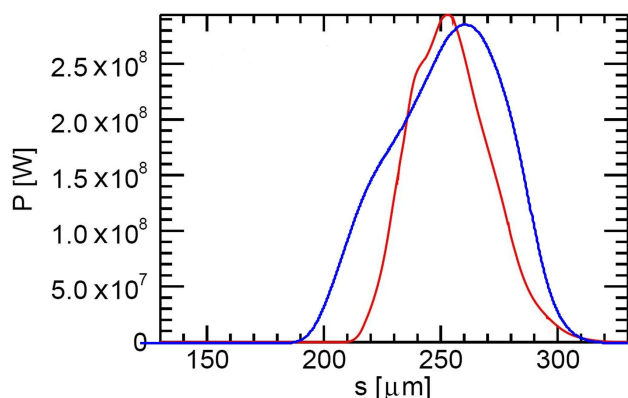


Figure 5: Comparison between FELO radiation power output pulse (blue) with that of a Genesis simulation (red) approaching saturation of the output energy. The parameters are for 10 eV photon output.

peak powers are significantly greater than the predicted by the steady-state simulations [2]. A plot of the shape of the pulse output close to cavity resonance is shown in Fig. 6. This pulse shape is typical of superradiant radiation with a narrow, high peak power spike followed by secondary pulses or ringing [15].

CONCLUSIONS

The FELO code package has been developed to allow reasonably quick and accurate simulations of cavity FELs operating from the low to high gain. The package has been used successfully in the development of the design of two cavity FELs for the UK 4GLS project. Some of these simulation results were presented. The code including manual is freely available for general use [10].

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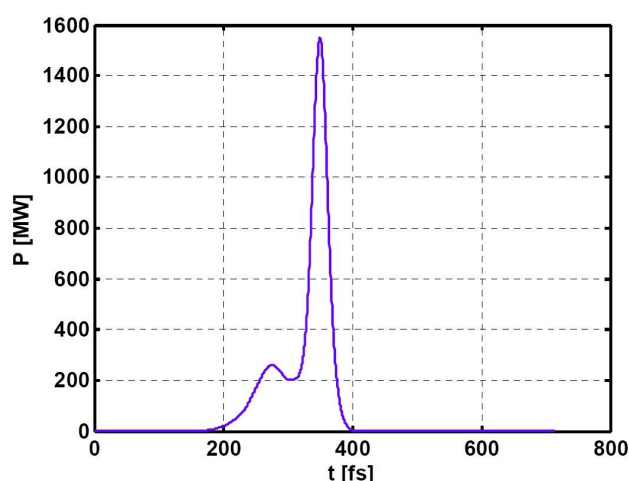


Figure 6: FELO simulation showing the 10 eV radiation power at saturation as a function of time for near-zero cavity detuning. The pulse shape demonstrates a spiking behaviour typical to FEL superradiance.

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